Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 15: Library of Functions

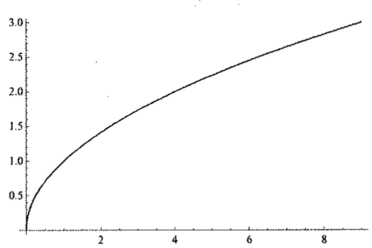
The goal of this lesson is to learn how to graph and evaluate basic functions. We know from Lesson 12 that the graph of a linear equation $y = mx + b$ is a straight line with slope m and y-intercept b. A linear function has a similar definition.

Definition 1. **Linear Functions:** *Let m and b be real numbers. The function $f(x) = mx + b$ is called a* ***linear function****. If $m = 0$, the function $f(x) = b$ is called a* ***constant function****.*

Definition 2. **Graphing Functions:** *Graph $f(x) = \sqrt{x}$.*

Solution: *To sketch the graph of $f(x) = \sqrt{x}$, we make a table of values.*

x	$f(x)$	(x, y)
0	$\sqrt{0} = 0$	$(0, 0)$
1	$\sqrt{1} = 1$	$(1, 1)$
4	$\sqrt{4} = 2$	$(4, 2)$
9	$\sqrt{9} = 3$	$(9, 3)$
16	$\sqrt{16} = 4$	$(16, 4)$



The Domain of $f(x) = \sqrt{x}$ is $[0, \infty)$, and its Range is $[0, \infty)$.

Example 1. **Example of a Piecewise Function:**

The function

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$$

is a piecewise function. Find $f(0)$ and $f(2)$.

Since $0 < 1$, then we use the first line, $f(x) = x^2$. So $f(0) = 0^2 = 0$. Because $2 > 1$, then we use the second line, $f(x) = 2x + 1$. So $f(2) = 2(2) + 1 = 5$.

Definition 3. **Greatest Integer Function:** *The* ***greatest integer function*** *is denoted by $f(x) = [[x]]$ = the greatest integer less than or equal to x. For example, $[[-3.7]] = -4$, $[[0.99]] = 0$, $[[2]] = 2$.*

EXAMPLE 4 **Graphing the Cube Root Function**

Graph $f(x) = \sqrt[3]{x}$.

SOLUTION

We use the point plotting method to graph $f(x) = \sqrt[3]{x}$. For convenience, we select values of x that are perfect cubes. See Table 2.9. The graph of $f(x) = \sqrt[3]{x}$ is shown in Figure 2.62.

TABLE 2.9

x	$y = f(x)$	(x, y)
-8	$\sqrt[3]{-8} = -2$	$(-8, -2)$
-1	$\sqrt[3]{-1} = -1$	$(-1, -1)$
0	$\sqrt[3]{0} = 0$	$(0, 0)$
1	$\sqrt[3]{1} = 1$	$(1, 1)$
8	$\sqrt[3]{8} = 2$	$(8, 2)$

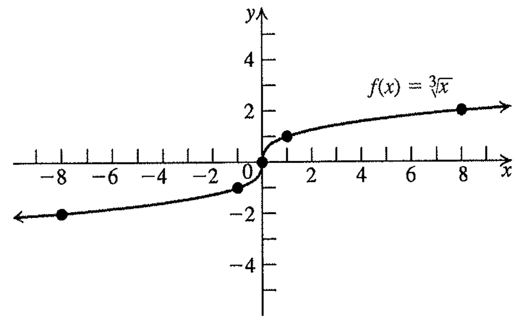


FIGURE 2.62 **Graph of $y = \sqrt[3]{x}$**

The domain of $f(x) = \sqrt[3]{x}$ is $(-\infty, \infty)$, and its range also is $(-\infty, \infty)$. ■ ■ ■

Practice Problem 4 Graph $g(x) = \sqrt[3]{-x}$ and find its domain and range. ■

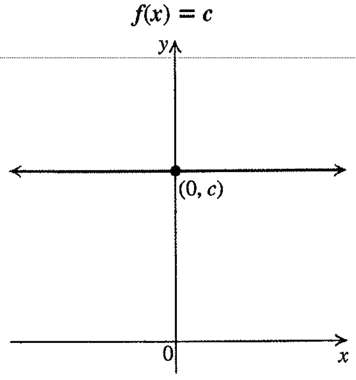
3 Graph additional basic functions.

Basic Functions

As you progress through this course and future mathematics courses, you will repeatedly come across a small list of basic functions. The following box lists some of these common algebraic functions, along with their properties. You should try to produce these graphs by plotting points and using symmetries. The unit length in all of the graphs shown is the same on both axes.

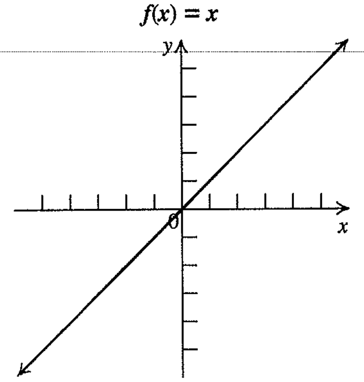
A LIBRARY OF BASIC FUNCTIONS

Constant Function
$f(x) = c$



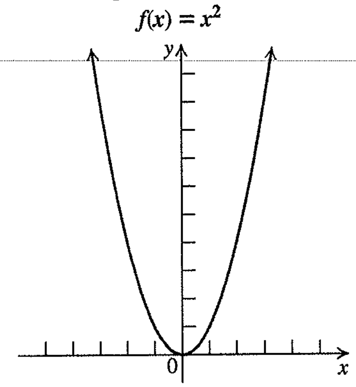
Domain: $(-\infty, \infty)$
Range: $\{c\}$
Constant on $(-\infty, \infty)$
Even function (y-axis symmetry)

Identity Function
$f(x) = x$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function (origin symmetry)

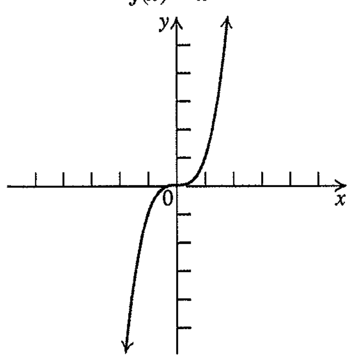
Squaring Function
$f(x) = x^2$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function (y-axis symmetry)

Cubing Function

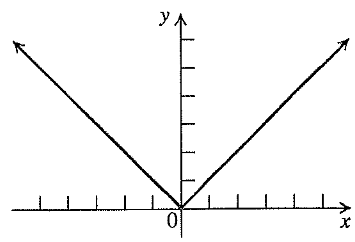
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function (origin symmetry)

Absolute Value Function

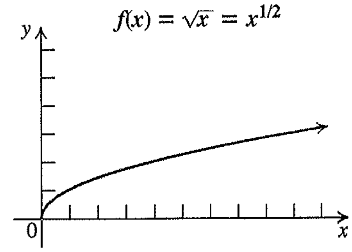
$f(x) = |x|$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function (y-axis symmetry)

Square Root Function

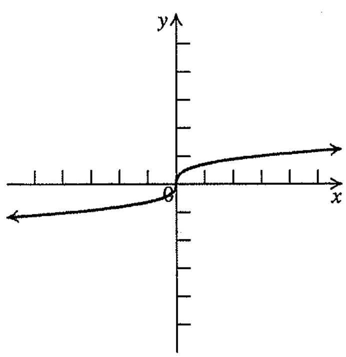
$f(x) = \sqrt{x} = x^{1/2}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Increasing on $(0, \infty)$
Neither even nor odd (no symmetry)

Cube Root Function

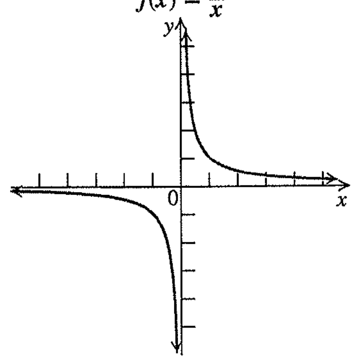
$f(x) = \sqrt[3]{x} = x^{1/3}$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function (origin symmetry)

Reciprocal Function

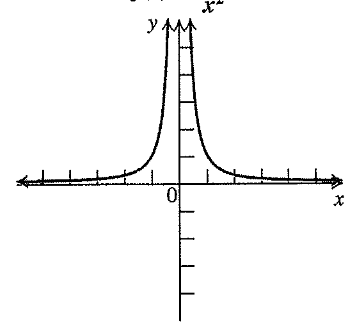
$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Decreasing on $(-\infty, 0) \cup (0, \infty)$
Odd function (origin symmetry)

Reciprocal Square Function

$f(x) = \frac{1}{x^2}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$
Increasing on $(-\infty, 0)$
Decreasing on $(0, \infty)$
Even function (y-axis symmetry)

4 Evaluate and graph piecewise functions.

Piecewise Functions

In the definition of some functions, different rules for assigning output values are used on different parts of the domain. Such functions are called **piecewise functions.** For example, in Peach County, Georgia, a section of the interstate highway has a speed limit of 55 miles per hour (mph). If you are caught speeding between 56 and 74 mph, your fine is \$50 plus \$3 for every mile per hour over 55 mph. For 75 mph and higher, your fine is \$150 plus \$5 for every mile per hour over 75 mph.

Let $f(x)$ be the piecewise function that represents your fine for speeding at x miles per hour. We express $f(x)$ as a piecewise function.

$$f(x) = \begin{cases} 50 + 3(x - 55), & 56 \le x < 75 \\ 150 + 5(x - 75), & x \ge 75 \end{cases}$$